HONORS GEOMETRY CHAPTER 2 WORKBOOK

FALL 2016

Chapter 2 Miscellaneous: The Structure of Geometry

Vocabulary	Definition	Example
Deductive Structure		**Elements:** **1.** **2.** **3.** **4.**
Postulate (axiom)		
Definitions		**Example:** **Reversed:**

Theorems and postulates are **<u>not</u>** always reversible.
For example, True or False: If two angles are right angles, then they are congruent.

The converse statement, True or False: If two angles are congruent, then they are right angles.

Types of Statements	
Conditional Statement: if ____ then ____	**Converse Statements: if _____ then _____**
Inverse Statement: if _____ then ______	**Contrapositive Statement: if ____ then ____**

Example 1: Write the converse to the following statement: "If an angle is a right angle, then it is 90°."

Example 2: Write the contrapositive of the following statement: "If a polygon is a square, then it is a rectangle."

Definitions are **ALWAYS REVERSIBLE!**
Example:

Theorems and Postulates: **not always** reversible.
Example:

**When we start writing proofs, you must memorize postulates, definitions, and theorems in order to be successful.*

Vocabulary	Definition	Example
Venn Diagrams		

Example 4: "If Jenny lives in Oswego, then Jenny lives in Illinois."
We will assume this conditional is true.

- Now, consider the converse:

  Is the converse true or false?

- Now, consider the contrapositive:

  Is the contrapositive true or false?

Draw:

Vocabulary	Definition
Theorem	
Chains of Reasoning	

Example 5: Use the following statements to make a concluding statement.

1. "If you study hard, then you will earn a good grade." ($p \rightarrow q$)
2. "If you earn a good grade, then your family will be happy." ($q \rightarrow r$)

Honors Geometry
Chapter 2 Misc. Worksheet

Write each statement in if-then form.

1. A polygon with four sides is a quadrilateral.

2. An acute angle has a measure less than 90.

Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample.

3. If you have five dollars, then you have five one-dollar bills.

4. If I roll two six-sided dice and sum of the numbers is 11, then one die must be a five.

5. If two angles are supplementary, then one of the angles is acute.

6. Write the converse, inverse, and contrapositive of the conditional statement.
Determine whether each statement is true or false. If a statement is false, find a counterexample.

If 89 is divisible by 2, then 89 is an even number.

For each statement, draw a Venn diagram. Then write the sentence in if-then form.

7. Every dog has long hair.

8. All rational numbers are real.

For each statement, draw a Venn diagram. Then write the sentence in if-then form.

9. People who live in Iowa like corn.

10. Staff members are allowed in the faculty lounge.

Use the Chains of Reasoning to draw a valid conclusion from each set of statements, if possible. If no valid conclusion can be drawn, write *no valid conclusion*.

11. If two angles form a linear pair, then the two angles are supplementary.
If two angles are supplementary, then the sum of their measures is 180.

12. If a hurricane is Category 5, then winds are greater than 155 miles per hour.
If winds are greater than 155 miles per hour, then trees, shrubs, and signs are blown down.

Section 2.2 Notes: Logic

Statement	Definition	Symbols
Negation		
Conjunction		
Disconjunction		

A convenient method for organizing the truth values of statements is to use a ______________. Truth tables can be used to determine truth values of negations and compound statements.

Example 1: Construct a truth table for $\sim p \vee q$.

Step 1 Make columns with the heading p, q, $\sim p$, and $\sim p \vee q$.

Step 2 List the possible combinations of truth values for p and q.

Step 3 Use the truth values of p to determine the truth values of $\sim p$.

Step 4 Use the truth values of $\sim p$ and q to write the truth values for $\sim p \vee q$.

p	q	$\sim p$	$\sim p \vee q$

Example 2: Construct a truth table for $p \vee (\sim q \wedge r)$.

p	q	r	$\sim q$	$\sim q \wedge r$	$p \vee (\sim q \wedge r)$

Example 3: Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value.

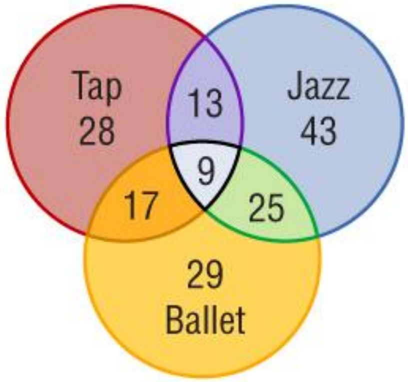
p: -3 - 2 = -5
q: Vertical angles are congruent.
r: 2 + 8 > 10
s: The sum of the measures of complementary angles is 90.

a. $p \wedge \sim q$

b. p or s

Venn Diagrams: Conjunctions can be illustrated with Venn diagrams.

Example 4: The Venn diagram shows the number of students enrolled in Monique's Dance School for tap, jazz, and ballet classes.

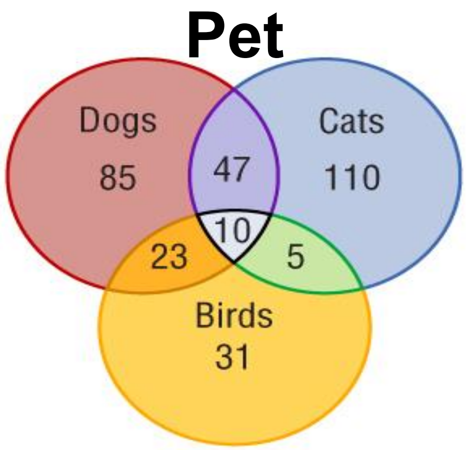


a) How many students are enrolled in all three classes?

b) How many students are enrolled in tap or ballet?

c) How many students are enrolled in jazz and ballet, but not tap?

Example 5: The Venn diagram shows the number of students at Manhattan School that have dogs, cats, and birds as household pets.

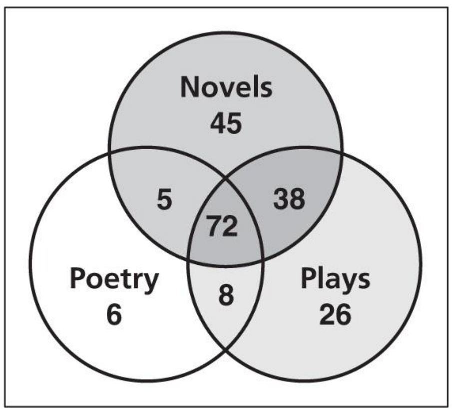


a) How many students in Manhattan School have a dog, a cat, or a bird?

b) How many students have dogs or cats?

c) How many students have dogs, cats, and birds as pets?

Example 6: Two hundred people were asked what kind of literature they like to read. They could choose among novels, poetry, and plays. The results are shown in the Venn diagram.



a. How many people said they like all three types of literature?

b. How many like to read poetry?

c. What percentage of the people who like plays also like novels and poetry?

Honors Geometry
2.2 Worksheet

Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value.

p: 60 seconds = 1 minute
q: Congruent supplementary angles each have a measure of 90.
r: $-12 + 11 < -1$

1. $p \wedge q$

2. $q \vee r$

3. $\sim p \vee q$

4. $\sim p \wedge \sim r$

Complete each truth table.

5.

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T			
T	F			
F	T			
F	F			

6.

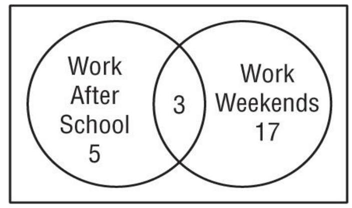
p	q	$\sim p$	$\sim p \vee q$	$p \wedge (\sim p \vee q)$
T	T			
T	F			
F	T			
F	F			

Construct a truth table for each compound statement.

7. $q \vee (p \wedge \sim q)$

8. $\sim q \wedge (\sim p \vee q)$

9. SCHOOL The Venn diagram shows the number of students in the band who work after school or on the weekends.



a. How many students work after school and on weekends?

b. How many students work after school or on weekends?

Honors Geometry
2.2 Textbook Homework

pg. 111-114 #16, 17, 35-38, 59-61

Common Core State Standards
G.MG.3 Apply geometric methods to solve problems.

Student Learning Targets
1. Students will be able to identify and use basic postulates about points, lines and planes.
2. Students will be able write paragraph proofs.

Section 2.5 Notes: Postulates and Paragraph Proofs

Vocabulary	Definition	Real Life Example
Postulate		

Points, Lines, and Planes Postulates		
Postulate	**Definition**	**Picture**
2.1		
2.2		
2.3		
2.4		
2.5		
Intersection of Lines and Planes		
2.6		
2.7		

Example 1: Explain how the picture illustrates that the statement is true. Then state the postulate that can be used to show the statement is true.

a) Lines *n* and *l* intersect at point K.

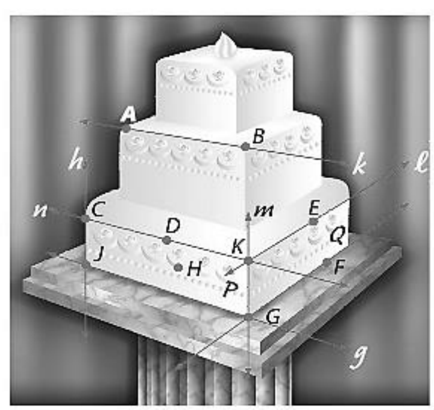
b)Planes **P** and **Q** intersect in line *m*.

c) Points D, K, and H determine a plane.

d) Point D is also on the line *n* through points C and K.

e) Points D and H are collinear.

f) Points E, F and G are coplanar.



You can use postulates to explain your reasoning when analyzing statements.

Example 2: Determine whether the following statement is *always*, *sometimes*, or *never* true. Explain.

a) If plane T contains $\overleftrightarrow{EF}$ and $\overleftrightarrow{EF}$ contains point G, then plane T contains point G.

b) $\overleftrightarrow{GH}$ contains three noncollinear points.

Vocabulary	**Definition**	**Real Life Example**
Proof		
Theorem		

The Proof Process	
Step 1:	
Step 2:	
Step 3:	
Step 4:	
Step 5:	

Example 3: Use the steps above to prove the given statement. Given that M is the midpoint of $\overline{XY}$. Write a paragraph proof to show that $\overline{XM} \cong \overline{MY}$.

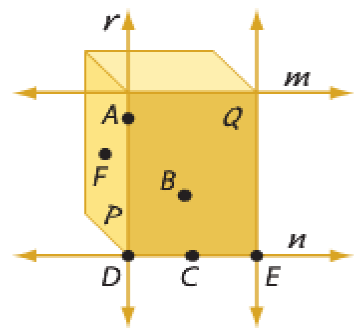
The conjecture in Example 3 is known as the Midpoint Theorem.

Vocabulary	Definition	Picture
Midpoint Theorem		

**Once a conjecture has been proven, it can be stated as a theorem and used in other proofs.*
-Now that we have proven the midpoint theorem, we can use it in other proofs.

Summary

Explain how the figure illustrates that each statement is true. Then state the postulate that can be used to show each statement is true.



1. Planes P and Q intersect in line r.

2. Lines r and n intersect at point D.

3. Lines n contains points C, D, and E.

4. Plane P contains the points A, F, and D.

5. Line n lies in plane Q.

6. line r is the only line through points A and D.

Determine whether each statement is *always, sometimes* or *never* true. Explain your reasoning.

7. The intersection of three planes is a line.

8. Line r contains only point P.

9. Through two points, there is exactly one line.

ERROR ANALYSIS Omari and Lisa were working on a paragraph proof to prove that if $\overline{AB}$ is congruent to $\overline{BD}$ and A, B, and D are collinear, then B is the midpoint of $\overline{AD}$. Each student started his or her proof in a different way. Is either of them correct? Explain your reasoning.

Omari	Lisa
If B is the midpoint of $\overline{AB}$, then B divides $\overline{AD}$ into two congruent segments.	$\overline{AB}$ is congruent to $\overline{BD}$ and A, B, and D are collinear.

Honors Geometry

2.5 Textbook Homework

pg. 131 –133: #24 – 30 even, 36, 38, & 45

Common Core State Standards +
G.CO.9 Prove theorems about lines and angles.

Student Learning Targets
1. Students will be able to use algebra to write two-column proofs.

Section 2.6 Notes: Algebraic Proof

Properties of Real Numbers	Symbols
Addition Property of Equality	
Subtraction Property of Equality	
Multiplication Property of Equality	
Division Property of Equality	
Reflexive Property of Equality	
Symmetric Property of Equality	
Transitive Property of Equality	
Substitution Property of Equality	
Distributive Property	

Vocabulary	Definition
Algebraic Proof	

Example 1: Justify each step when solving an equation.

a) Solve $2(5-3a)-4(a+7)=92$.

b) Solve $-3(a+3)+5(3-a)=-50$.

Example 2:

a) **SCIENCE** If the distance d an object travels is given by $d=20t+5$, the time t that the object travels is given by $t=\frac{d-5}{20}$. Write a two-column proof to verify this conjecture.

Begin by stating what is given and what you are to prove.

Given: $d=20t+5$

Prove: $t=\frac{d-5}{20}$

Statements	Reasons
1. $d = 20t + 5$	1. Given
2. $d - 5 = 20t$	2.
3. $\frac{d-5}{20} = t$	3.
4. $t = \frac{d-5}{20}$	4.

b) Which of the following statements would complete the proof of this conjecture?

If the formula for the area of a trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$, then the height h of the trapezoid is given by $h = \frac{2A}{(b_1 + b_2)}$.

Given: $A = \frac{1}{2}(b_1 + b_2)h$

Prove: $h = \frac{2A}{(b_1 + b_2)}$

Statements	Reasons
1. $A = \frac{1}{2}(b_1 + b_2)h$	1. Given
2.	2. Multiplication Property of Equality
3. $\frac{2A}{\left(b_1 + b_2\right)} = h$	3.
4. $h = \frac{2A}{(b_1 + b_2)}$	4. Symmetric Property of Equality

Example 3: Write a Geometric Proof

If $\angle A \cong \angle B$, $m\angle B = 2\left(m\angle C\right)$, and $m\angle C = 45°$, then $m\angle A = 90°$. Write a two-column proof to verify this conjecture.

Given: $\angle A \cong \angle B$, $m\angle B = 2\left(m\angle C\right)$,
$m\angle C = 45°$

Prove: $m\angle A = 90°$

Statements	Reasons
1. $\angle A \cong \angle B$, $m\angle B = 2\left(m\angle C\right)$, $m\angle C = 45°$	1. Given
2. $m\angle A = m\angle B$	2.
3. $m\angle A = 2\left(m\angle C\right)$	3.
4. $m\angle A = 2\left(45\right)$	4.
5. $m\angle A = 90°$	5.

Honors Geometry

2.6 Textbook Homework **pg. 140-143 #17, 18, 42, 46-49**

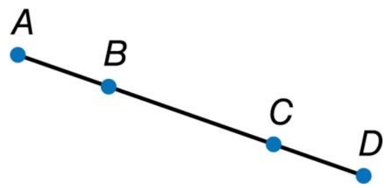
Common Core State Standards
G.CO.9 Prove theorems about lines and angles.

Student Learning Targets
1. Students will be able to use algebra to write two-column proofs.
2. Students will be able to use properties of equality to write geometric proofs.

Section 2.7 Notes: Proving Segment Relationships

Vocabulary	Definition	Picture
Ruler Postulate		
Segment Addition Postulate		

Example 1: Prove that if $\overline{AB} \cong \overline{CD}$, then $\overline{AC} \cong \overline{BD}$.



Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.

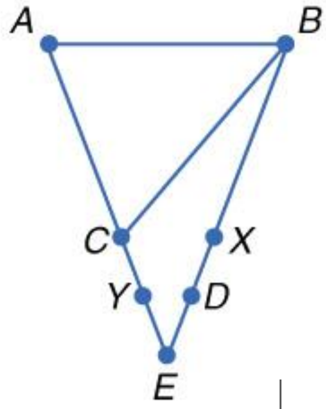
Example 2: Prove the following

Given: $AC = AB$

$AB = BX$

$CY = XD$

Prove: $AY = BD$



Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.

Properties of Segment Congruence	
Reflexive Property of Congruence	
Symmetric Property of Congruence	
Transitive Property of Congruence	

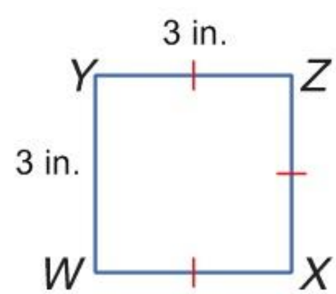
Example 3: BADGE Jamie is designing a badge for her club. The length of the top edge of the badge is equal to the length of the left edge of the badge. The top edge of the badge is congruent to the right edge of the badge, and the right edge of the badge is congruent to the bottom edge of the badge. Prove that the bottom edge of the badge is congruent to the left edge of the badge.

Given: $WY = YZ$

$\overline{YZ} \cong \overline{XZ}$

$\overline{XZ} \cong \overline{WX}$

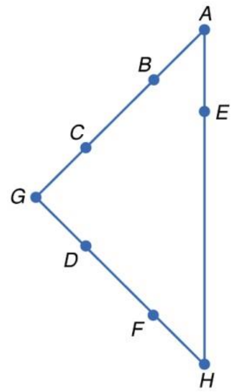
Prove: $\overline{WX} \cong \overline{WY}$



Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

Example 4: Prove the following.

Given: $\overline{GD} \cong \overline{BC}$
$\overline{BC} \cong \overline{FH}$
$\overline{FH} \cong \overline{AE}$
Prove: $\overline{AE} \cong \overline{GD}$



Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

Honors Geometry
2.7 Textbook Homework

pg. 147-149 #4, 10, 12, 17, & 20

Common Core State Standards
G.CO.9 Prove theorems about lines and angles.

Student Learning Targets
1. Students will be able to use algebra to write two-column proofs.
2. Students will be able to use properties of equality to write geometric proofs.

Section 2.8 Notes: Proving Angle Relationships

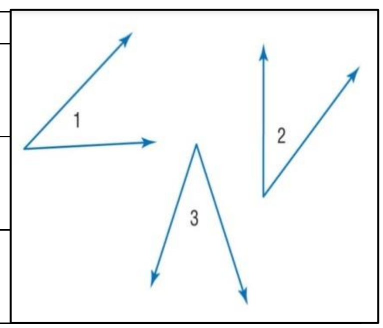
Vocabulary	Definition	Picture
Angle Addition Postulate		

Example 1: CONSTRUCTION Using a protractor, a construction worker measures that the angle a beam makes with a ceiling is 42°. What is the measure of the angle the beam makes with the wall?

Vocabulary	Definition	Picture
Supplement Theorem		
Complement Theorem		

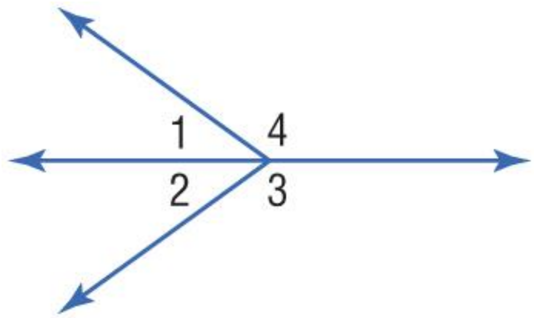
Example 2: TIME At 4 o'clock, the angle between the hour and minute hands of a clock is 120º. When the second hand bisects the angle between the hour and minute hands, what are the measures of the angles between the minute and second hands and between the second and hour hands?

Properties of Angle Congruence	
Reflexive Property of Congruence	
Symmetric Property of Congruence	
Transitive Property of Congruence	



Vocabulary	Definition	Picture
Congruent Supplement Theorem		
Congruent Complement Theorem		

Example 3: In the figure, ∠1 and ∠4 form a linear pair, and $m\angle 3 + m\angle 1 = 180°$. Prove that ∠3 and ∠4 are congruent.



Statements	Reasons
1.	1. Given
2. ∠1 and ∠4 form a linear pair	2.
3. ∠1 and ∠4 are supplementary	3.
4.	4. Def. of Supplementary Angles
5.	5.

Vocabulary	Definition	Picture
Vertical Angles Theorem		

Example 4: If ∠1 and ∠2 are vertical angles and $m\angle 1 = (d - 32)°$ and $m\angle 2 = (175 - 2d)°$, find $m\angle 1$ and $m\angle 2$. Justify each step.

Statements	Reasons
1.	1.Given
2. $\angle 1 \cong \angle 2$	2.
3. $m\angle 1 = m\angle 2$	3.
4.	4. Substitution
5.	5. Addition Property
6.	6.
7. $d = 69$	7.
8.	8.

Right Angle Theorems		
Theorem	**Definition**	**Picture**
2.9		
2.10		
2.11		
2.12		
2.13		

Honors Geometry
2.8 Textbook Homework

2.8 pg. 156-158 #10, 14, 20, 21, 27